Outstanding Reading

- Read all chapters assigned from IT-I (EE514, Winter 2012).
- Read chapter 8 in the book.
- Read chapter 9 in the book.
- Read chapter 10 in the book (chapter on rate distortion theory).
- Read chapter 14 in the book (Kolmogorov complexity).
- Read chapter 13, section on Lempel Ziv compression, in the book.
- Read chapter 15 in C&T.
- Read chapters 4, 5, 9, 10, 15 in El Gamal & Kim.
Announcements, Assignments, and Reminders

- Please do use our discussion board (https://catalyst.uw.edu/gopost/board/bilmes/27386/) for all questions, comments, so that all will benefit from them being answered.
On Final Presentations

- Your task is to give a 15-20 minute presentation that summarizes 2-3 related and significant papers that come from IEEE Transactions on Information Theory (or a very related area).
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- This is a real challenge and will require significant work! Many of the papers are complex. To get a good grade, you will need to work very hard to present very complex ideas in an extremely simple yet still precise way.
- Again, don’t expect this to be easy, you might need to try a few topics until you find one that is suitable.
Final Presentation Milestones

All submissions done in PDF file format via our dropbox (https://catalyst.uw.edu/collectit/dropbox/bilmes/21171)

- Final presentations: Monday, June 4th from 2:00pm - 5:00pm in EEB-417 (my lab).
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- Very important part of modern IT (still currently being actively researched).
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- The most general case (first). We have an arbitrary network:

\[ X_1, Y_1 \quad X_2, Y_2 \]
\[ X_3, Y_3 \]
\[ \cdots \]
\[ X_m, Y_m \quad X_4, Y_4 \]
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Each sender $X_i$ is trying to communicate simultaneously with each receiver $Y_i$ (i.e., for all $i$, $X_i$ is sending to $\{Y_i\}_i$)
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- The \( X_i \) are not necessarily independent.
Broadcast Channel (BC)

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Ex: cell-phone towers: separate message to each cell phone over essentially the same channel (lots of interference potential between messages).
Ex: classroom lecture to students (plural).

Goal is to provide information at an appropriate rate to both good (and bad (but of course there is no such thing), actually, it's all the fault of the professor and poorly prepared lectures!)
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(W_1, W_2) \rightarrow \text{Encoder} \rightarrow p(y_1, y_2|x) \rightarrow \text{Decoder} \rightarrow \hat{W}_1
\]

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Some definitions

- One sender, two receivers, sender has messages for both receivers, simultaneous broadcast.
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- The information for each receiver is independent.
- Channel: $(\mathcal{X}, \mathcal{Y}_1, \mathcal{Y}_2), p(y_1, y_2|x)$
- Memoryless: $\Pr(y_{1:n}^1, y_{1:n}^2|x_{1:n}) = \prod_{i=1}^{n} p(y_i^1, y_i^2|x_i)$. 

$P(n)e = \Pr[ g_1(Y_1^{1:n}) \neq W_1 \lor g_2(Y_2^{1:n}) \neq W_2]$ where $W_1, W_2$ are both uniformly distributed (and thus independent).
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- Code sequence: \(((2^{nR_1}, 2^{nR_2}), n)\) code for Broadcast channel with independent information consists of encoder:
  \[
  X : (\{1, 2, \ldots, 2^{nR_1}\} \times \{1, 2, \ldots, 2^{nR_2}\}) \rightarrow X^n
  \]
  (1)

  and two decoders:
  \[
  g_1 : Y_1^{1:n} \rightarrow \{1, \ldots, 2^{nR_1}\}, \quad g_2 : Y_2^{1:n} \rightarrow \{1, \ldots, 2^{nR_2}\}
  \]
  (2)

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- **Probability of error:**
  \[P_e^{(n)} = \Pr\left[(g_1(Y_{1:n}^1) \neq W_1) \lor (g_2(Y_{1:n}^2) \neq W_2)\right]\]  
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  where \(W_1, W_2\) are both uniformly distributed (and thus independent).
Achievable: A rate pair \((R_1, R_2)\) is said to be achievable for BC if there exists sequence of \(((2^{nR_1}, 2^{nR_2}), n)\) codes with \(P_e^{(n)} \to 0\).
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\[
X : ([2^{nR_0}] \times [2^{nR_1}] \times [2^{nR_2}]) \to \mathcal{X}^n
\]

and two decoders:

\[
g_1 : \mathcal{Y}_{1:n}^1 \to [2^{nR_0}] \times [2^{nR_1}], \quad g_2 : \mathcal{Y}_{1:n}^2 \to [2^{nR_0}] \times [2^{nR_2}] \quad (4)
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\]

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P_e(n) = \Pr [(g_1(Y_{1:n}^1) \neq (W_0, W_1)) \lor (g_2(Y_{1:n}^2) \neq (W_0, W_2))] \tag{5}
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where \(W_0, W_1, W_2\) are both uniformly distributed (and thus independent).
More Definitions

- **Achievable:** A rate pair \((R_1, R_2)\) is said to be achievable for BC if there exists a sequence of \(((2^n R_1, 2^n R_2), n)\) codes with \(P_e(n) \to 0\).
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- **Achievable:** A rate triple \((R_0, R_1, R_2)\) is said to be achievable for BC in a similar way.
**More Definitions**

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- **capacity region** is the closure of all achievable rates.
On Capacity Region

Theorem 2.1

The capacity region of achievable rates in a broadcast channel depends only on $p(y_1|x)$ and $p(y_2|x)$.

proof sketch.

- $Y_1$ (resp. $Y_2$) error depends only on $p(x, y_1)$ (resp. $p(x, y_2)$) not on $p(x, y_1, y_2)$.
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- $Y_1$ (resp. $Y_2$) error depends only on $p(x, y_1)$ (resp. $p(x, y_2)$) not on $p(x, y_1, y_2)$.
- $A \triangleq (g_1(Y_{1:n}^1) \neq W_1) \lor (g_2(Y_{1:n}^2) \neq W_2)$, $B_1 \triangleq (g_1(Y_{1:n}^1) \neq W_1)$, $B_2 \triangleq (g_2(Y_{1:n}^2) \neq W_2)$. 
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- Then \( B_1 \Rightarrow A \) and \( B_2 \Rightarrow A \), so \( p(B_1) \leq p(A) \) and \( p(B_2) \leq p(A) \).
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- Then \( B_1 \Rightarrow A \) and \( B_2 \Rightarrow A \), so \( p(B_1) \leq p(A) \) and \( p(B_2) \leq p(A) \).
- Hence \( \max \{ p(B_1), p(B_2) \} \leq p(A) \).
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- Then \( B_1 \Rightarrow A \) and \( B_2 \Rightarrow A \), so \( p(B_1) \leq p(A) \) and \( p(B_2) \leq p(A) \).
- Hence \( \max \{ p(B_1), p(B_2) \} \leq p(A) \).
- \( \max \{ P_{e}^1, P_{e}^2 \} \leq P_{e}^{1+2} \leq P_{e}^1 + P_{e}^2 \), so while \( 1 + 2 \) error depends on the full joint \( p(x, y_1, y_2) \), to drive down the \( 1 + 2 \) error, it is necessary and sufficient to drive down both \( P_{e}^1, P_{e}^2 \) individually.
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The capacity region of achievable rates in a broadcast channel depends only on $p(y_1|x)$ and $p(y_2|x)$.

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- Then $B_1 \Rightarrow A$ and $B_2 \Rightarrow A$, so $p(B_1) \leq p(A)$ and $p(B_2) \leq p(A)$.
- Hence $\max \{p(B_1), p(B_2)\} \leq p(A)$.
- $\max \{P_e^1, P_e^2\} \leq P_e^{1+2} \leq P_e^1 + P_e^2$, so while $1 + 2$ error depends on the full joint $p(x, y_1, y_2)$, to drive down the $1 + 2$ error, it is necessary and sufficient to drive down both $P_e^1, P_e^2$ individually.
- this essentially completes the proof
Simple Bounds on Capacity

Given a BC $p(y_1, y_2| x)$, define

$$C_j = \max_{p(x)} I(X; Y_j)$$

for $j = 1, 2$, capacities of DMCs $p(y_1|x)$ and $p(y_2|x)$
Simple Bounds on Capacity

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  capacities of DMCs
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- These give us a time-sharing “inner bound” of the capacity region of a BC.
Simple Bounds on Capacity

Given a BC \( p(y_1, y_2 | x) \), define
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These give us a time-sharing “inner bound” of the capacity region of a BC.

On the other hand, \( R_1 + R_2 \leq C_{12} = \max_p(x) I(X; Y_1, Y_2) \) gives us outer bound.
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Neither bound is tight in general.
Simple Bounds on Capacity

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Superposition Coding Inner Bound

- As mentioned, one receiver may be cleaner (or stronger) than other.
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- As an example, consider the following pair of BSCs. I.e., \( \text{BSC}(p_1) \) and \( \text{BSC}(p_2) \) where w.l.o.g., \( p_1 < p_2 < 1/2 \).
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- The weaker receiver recovers its intended message, and the stronger receiver recovers its intended message using successive cancellation decoding (as we saw for the MAC channel).
- As an example, consider the following pair of BSCs. I.e., BSC($p_1$) and BSC($p_2$) where w.l.o.g., $p_1 < p_2 < 1/2$.
- This can be seen as a BC as in the following, inner bound on right:

$$Z_1 \sim \text{Bernoulli}(p_1)$$

$$Z_2 \sim \text{Bernoulli}(p_2)$$

$$X \longrightarrow + \longrightarrow Y_1$$

$$Y_1' = Y_1 + Z_1$$

$$Y_2' = Y_2 + Z_2$$

$$R_1 = 1 - H(p_2)$$

$$R_2 = 1 - H(p_1)$$

Time division

inner bound
Superposition Coding Inner Bound

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- Let $X = U + V \pmod{2}$ (i.e., $0 + 0 = 0$, $0 + 1 = 1$, $1 + 1 = 0$), and define $\oplus$ as componentwise mod 2 addition of vectors or scalars (thus $X = U \oplus V$).
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- Choose rates $R_1$ and $R_2$.

- Code generation: randomly generate $|W_2| = 2^{nR_2}$ sequences of the form $u_{1:n}(w_2)$ for each $w_2 \in [2^{nR_2}]$, based on Bernoulli(1/2). These will act as “cloud centers” as we will see.
Superposition Coding Inner Bound

Further code generation: randomly generate $2^{nR_1}$ sequences $v_1:n(w_1)$ for each $w_1 \in [2^{nR_1}]$, here generated Bernoulli($\alpha$).
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- Further code generation: randomly generate $2^{nR_1}$ sequences $v_{1:n}(w_1)$ for each $w_1 \in [2^{nR_1}]$, here generated Bernoulli($\alpha$).

- To broadcast message $(w_1, w_2)$ the sender sends $x_{1:n}(w_1, w_2) \triangleq u_{1:n}(w_2) \oplus v_{1:n}(w_1)$. 

Receiver 2: needs to recover $w_2$, and has access to $y_{2:1:n} = u_{1:n}(w_2) \oplus v_{1:n}(w_1) \oplus z_{2:1:n}$ where $z_{2:1:n}$ is the BSC channel noise, and moreover via the second grouping, we can consider $(v_{1:n}(w_1) \oplus z_{2:1:n})$ as greater noise.

Thus, for receiver 2, message destined for receiver 1 is seen as noise. $P(e_2) \to 0$ as $n \to \infty$ as long as $R_2 < 1 - H(\alpha^* p_2)$ where $\alpha^* p_2 = \alpha (1-p_2) + (1-\alpha) p_2$ is the noise rate of the "effective" BSC between sender and receiver 2.
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$P_{e2}^{(n)}$, probability of error for receiver 2 $\to 0$ as $n \to \infty$ as long as $R_2 < 1 - H(\alpha \ast p_2)$ where $\alpha \ast p_2 \triangleq \alpha(1 - p_2) + (1 - \alpha)p_2$ is the noise rate of the “effective” BSC between sender and receiver 2.
Now, for receiver 1 (the cleaner one), we use successive cancellation decoding (as we did for the MAC).

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\begin{align*}
&\text{where } z_1 \text{ is the BSC channel noise for receiver 1.}
&\text{We can use this to recover } w_2 \\
&\text{treating } v_1 \text{ as part of the noise (but presumably receiver 1's rate can handle this).}
&\text{Given } w_2 \text{ we have access to } u_1 \text{ and can form:}
&\hat{y}_1 = u_1 \oplus v_1 \oplus z_1
&\text{which is a cleaner signal for receiver 1}
&\text{We then decode } \hat{y}_1 \text{ to recover } w_1.
\end{align*}
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Now, for receiver 1 (the cleaner one), we use successive cancellation decoding (as we did for the MAC).

Receiver 1: needs to recover $w_1$, and has access to

$$y_{1:n}^1 = u_{1:n}(w_2) \oplus v_{1:n}(w_1) \oplus z_{1:n}^1 = u_{1:n}(w_2) \oplus (v_{1:n}(w_1) \oplus z_{1:n}^1)$$

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We then decode $\hat{y}_1^n$ to recover $w_1$. 

Probability of error $\to 0$ as long as $R_1 \leq I(X;Y_1) = I(V;V \oplus Z_1^n) = H(\alpha^* p_1) - H(p_1)$ and $R_2 < 1 - H(\alpha^* p_1)$, the later condition already being satisfied since $p_1 < p_2 < 1/2$. 

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Prof. Jeff Bilmes
EE515A/Spring 2012/Info Theory – Lecture 37 - June 1st, 2012
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**Superposition Coding Inner Bound**

- Prof. Jeff Bilmes
Superposition Coding Inner Bound

- This leads to a better inner bound for \((R_1, R_2)\) of the form

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R_1 \leq H(\alpha \ast p_1) - H(p_1) \\
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for some \(\alpha \in [0, 1/2]\)
Superposition Coding Inner Bound

- This leads to a better inner bound for \((R_1, R_2)\) of the form

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R_1 \leq H(\alpha \cdot p_1) - H(p_1) \tag{6}
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\[
R_2 \leq 1 - H(\alpha \cdot p_2) \tag{7}
\]

for some \(\alpha \in [0, 1/2]\)

- This bound is larger than time-sharing.

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\begin{align*}
R_1 &\leq H(\alpha \cdot p_1) - H(p_1) \\
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\end{align*}
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We can view this as clouds, we have $2^{nR_2}$ “clouds” and within each cloud lies $2^{nR_1}$ codewords. We use $U$ (more precisely $u_{1:n}(w_2)$) to indicate cloud id.
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Receiver 1 can distinguish both cloud id and codeword within cloud, and does so by first identifying the cloud id, and then the codeword within the cloud.
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There is a tradeoff: We can fit more clouds in the large circle (increasing the rate $R_2$) but then the cloud radius would need to decrease (decreasing the rate $R_1$).
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I.e., there exists tradeoff between size of clouds and number of possible clouds.
Superposition Coding Inner Bound Theorem

Theorem 3.1

A rate pair \((R_1, R_2)\) is achievable for the discrete memoryless BC \(p(y_1, y_2|x)\) if:

\[
R_1 < I(X; Y_1|U) \quad \text{(8)}
\]
\[
R_2 < I(U; Y_2) \quad \text{(9)}
\]
\[
R_1 + R_2 < I(X; Y_1) \quad \text{(10)}
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for some pmf \(p(u, x)\).

- Note: \(U\) is an auxiliary random variable that does not correspond to the channel variables.
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- **Unlike time sharing variable before (which helped us to mix different rates), here \(U\) is more important in establishing the capacity region.**
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- Note: \(U\) is an auxiliary random variable that does not correspond to the channel variables.
- Unlike time sharing variable before (which helped us to mix different rates), here \(U\) is more important in establishing the capacity region.
- Also, note that we establish this bound for joint \(p(u, x)p(y_1, y_2|x)\), i.e., \(U \rightarrow X \rightarrow (Y_1, Y_2)\).
Proof of Theorem

Proof of Theorem 3.1.

- We first prove achievability (i.e., if rates satisfy above equations, \( \exists \) code that has vanishing error probability as \( n \to \infty \)).
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- **Decoder 2 claims** $\hat{w}_2$ if $(u_{1:n}(\hat{w}_2), y_{1:n}^2) \in A^{(n)}_\epsilon$ otherwise error.
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- **Symmetry again allows us to assume** $(w_1, w_2) = (1, 1)$.
Proof of Theorem

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- Error for decoder 2 decompose of:

$$E_{21} = \left\{ (U_{1:n}(w), Y_{1:n}^2) \notin A_{\epsilon}^{(n)} \right\}, \quad (11)$$

$$E_{22} = \left\{ (U_{1:n}(w_2), Y_{1:n}^2) \in A_{\epsilon}^{(n)} \text{ for some } w_2 \neq 1 \right\} \quad (12)$$
Proof of Theorem

proof of Theorem 3.1.

• Thus, decoder error is upper bounded by

\[ \Pr(E_2) \leq \Pr(E_{21}) + \Pr(E_{22}) \]  \hspace{1cm} (13)
Proof of Theorem

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- Next, error for decoder 1. Three types of error:

\[ E_{11} = \left\{ (U_{1:n}(1), X_{1:n}(1,1), Y_{1:n}^1) \notin A^{(n)}_\epsilon \right\} \]  \hspace{1cm} (14)

\[ E_{12} = \left\{ (U_{1:n}(1), X_{1:n}(w_1,1), Y_{1:n}^1) \in A^{(n)}_\epsilon \text{ for some } w_1 \neq 1 \right\} \]  \hspace{1cm} (15)

\[ E_{13} = \left\{ (U_{1:n}(w_2), X_{1:n}(w_1,w_2), Y_{1:n}^1) \in A^{(n)}_\epsilon \text{ for some } (w_1, w_2) \neq (1,1) \right\} \]  \hspace{1cm} (16)
Proof of Theorem

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- Again, $\Pr(E_1) \leq \Pr(E_{11}) + \Pr(E_{12}) + \Pr(E_{13})$,
- the first $\to 0$,
- the second $\to 0$ if $R_1 < I(X; Y_1|U) - \epsilon$,
- and the third $\to 0$ if $R_1 + R_2 < I(U, X; Y_1) - \epsilon = I(X; Y) - \epsilon$
  (recall $U \to X \to Y_1$ is a Markov chain).
Discussion

- Note that we can switch receivers to get the following form of bound as well:

\[
R_1 < I(U; Y_1) \quad (17)
\]
\[
R_2 < I(X; Y_2|U) \quad (18)
\]
\[
R_1 + R_2 < I(X; Y_2) \quad (19)
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- Superposition decoding is optimal for several classes of broadcast channel where one receiver is stronger than the other (e.g., Degraded broadcast channel that we briefly saw last time and that we visit again below),
Discussion

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Superposition decoding is optimal for several classes of broadcast channel where one receiver is stronger than the other (e.g., Degraded broadcast channel that we briefly saw last time and that we visit again below),

but there are cases where asking one receiver to decode both messages can strain the overall rates (see Chapter 8 in Gamal & Kim for example).
Degraded Broadcast channel

- Recall, no privacy, so $Y_1$ and $Y_2$ are both receiving all messages (i.e., the messages intended for each of $Y_1$ and $Y_2$).
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- In general, one of $Y_1$ and $Y_2$ will be noisy, assume w.l.o.g. that $Y_1$ is no noisier than $Y_2$. 

Degraded broadcast channel. A BC is physically degraded if:

$$p(y_1, y_2 | x) = p(y_1 | x) p(y_2 | y_1) \tag{20}$$

and stochastically degraded if

$$p(y_2 | y_1) = \sum_{y_1} p(y_1 | x) p'(y_2 | y_1) \tag{21}$$

in either case, same capacity since

$$I(X; Y_1, Y_2) = I(X; Y_1)$$
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and stochastically degraded if $\exists p'(y_2 | y_1)$ such that

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in either case, same capacity since $I(X; Y_1, Y_2) = I(X; Y_1)$. 

Prof. Jeff Bilmes
The capacity region for sending independent information over the degraded broadcast channel $X \rightarrow Y_1 \rightarrow Y_2$ is the convex hull of the closure of all $(R_1, R_2)$ satisfying:

\begin{align}
R_2 &\leq I(U; Y_2) \tag{22} \\
R_1 &\leq I(X; Y_1 | U) \tag{23} \\
R_1 + R_2 &\leq I(X; Y_1) \tag{24}
\end{align}

for some joint distribution $p(u)p(x|u)p(y_1, y_2|x)$ where the auxiliary variable $U$ has cardinality bounded by $|U| \leq \min \{|X|, |Y_1|, |Y_2|\}$. 
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- Note, $U$ is a “convenience” variable to show how the rates couple.
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- Also, the third equation $R_1 + R_2 \leq I(X; Y_1)$ is vacuous since from the first two, we can get:

\[
R_1 + R_2 \leq I(X; Y_1 | U) + I(U; Y_2) \tag{25}
\]

\[
\leq I(X; Y_1 | U) + I(U; Y_1) \quad \text{since } U \rightarrow X \rightarrow Y_1 \rightarrow Y_2 \tag{26}
\]

\[
= I(Y_1; X, U) \tag{27}
\]

\[
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  &= I(Y_1; X, U) \\
  &= I(Y_1; X) + I(Y_1; U | X) \\
  &= I(Y_1; X)
  \end{align*}  

  (25)  

  (26)  

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- Thus, the first two constitute a tighter upper bound which is traded off by the $U$ random variable and how $U$ relates to $X$ via $p(x|u)$. 

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- Thus, the first two constitute a tighter upper bound which is traded off by the $U$ random variable and how $U$ relates to $X$ via $p(x|u)$.
- Since we already have achievability for Superposition decoding, we immediately have achievability for degraded broadcast channel as well.
proof of converse of Theorem 3.2.

- Goal: A given sequence $(2^{nR_1}, 2^{nR_2}, n)$ codes with $P_e(n) \to 0$ must have $R_1 \leq I(X; Y_1|U)$ and $R_2 \leq I(U; Y_2)$ for some $p(u, x)$ such that $U \to X \to (Y_1, Y_2)$ is a Markov chain.
Degraded Channel: Converse

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- **Key:** is to be able to identify \(U\), the auxiliary variable.
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- Key is to be able to identify $U$, the auxiliary variable.

- Every $(2^{nR_1}, 2^{nR_2}, n)$ code induces joint distribution of form:

\[
p(w_1, w_2, x_{1:n}, y_{1:n}^1, y_{1:n}^2) = 2^{-n(R_1+R_2)} p(x_{1:n}|w_1, w_2) \prod_{i=1}^{n} p(y_i^1, y_i^2|x_i)
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\]

- **Fano** gives us:

\[
H(W_1|Y_{1:n}^1) \leq nR_1 P_{e}^{(n)} + 1 \leq n\epsilon_n, \tag{30}
\]

\[
H(W_2|Y_{1:n}^2) \leq nR_2 P_{e}^{(n)} + 1 \leq n\epsilon_n \tag{31}
\]

...
proof of converse of Theorem 3.2.

This then gives:

\[ nR_1 \leq I(M_1; Y_{1:n}^1) + n\epsilon, \quad \text{and} \quad nR_2 \leq I(M_2; Y_{1:n}^2) + n\epsilon \quad (32) \]
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- Since \( U \) represents \( W_2 \) in superposition scheme, we can choose \( U = W_2 \) and the Markov chain becomes \( U \rightarrow X_i \rightarrow (Y_i^1, Y_i^2) \).
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The proof proceeds similar to the above, but the key is how to identify \( U \) and this is done in a variety of ways (along with AEP like arguments).

See Gamal & Kim for details.
We can define a Gaussian broadcast channel and physically degraded version as follows:
Theorem 3.3

The capacity region for the Gaussian BC is the set of rate pairs \((R_1, R_2)\) such that

\[
R_1 \leq C(\alpha S_1) \quad \text{(33)}
\]

\[
R_2 \leq C \left( \frac{(1 - \alpha) S_2}{\alpha S_2 + 1} \right) \quad \text{(34)}
\]

for some \(\alpha \in [0, 1]\) where \(C(x) = \frac{1}{2} \log(1 + x)\) is the Gaussian capacity function.
Each sender $X_i$ is trying to communicate simultaneously with each receiver $Y_i$ (i.e., for all $i$, $X_i$ is sending to $\{Y_i\}_{i}$).
General Networks

Each sender $X_i$ is trying to communicate simultaneously with each receiver $Y_i$ (i.e., for all $i$, $X_i$ is sending to $\{Y_i\}_i$).

The $X_i$ are not necessarily independent.
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Rates from $i$ to $j$ are $R^{(i \rightarrow j)}$ to send message $W^{(i \rightarrow j)} \in \left\{1, 2, \ldots, 2^n R^{(i \rightarrow j)}\right\}$. 
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Rates from $i$ to $j$ are $R^{(i \to j)}$ to send message $W^{(i \to j)} \in \{1, 2, \ldots, 2^{nR^{(i \to j)}}\}$.

Each encoder $i$ gets full message $W^{i,1:m}$ and received history $Y^{(i)}_{1:k-1}$. 
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Each decoder gets all received symbols and all sent messages.
**General Networks**

### Theorem 4.1

If all rates \( \{ R^{(i \rightarrow j)} \}_{ij} \) are achievable, then there exists a joint distribution \( p(x^{(1:m)}) \) such that

\[
\forall S \subseteq V, \quad \sum_{i \in S, j \in V \setminus S} R^{(i \rightarrow j)} \leq I(X^S; Y^{V \setminus S} | X^{V \setminus S}) \tag{35}
\]

meaning, all information flow across all cuts is bounded by the conditional mutual information.
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meaning, all information flow across all cuts is bounded by the conditional mutual information.*

- **Note** \( I(X^S; Y^{V \setminus S} | X^{V \setminus S}) = H(X^V) + H(Y^{V \setminus S}, X^{V \setminus S}) - H(X^{V \setminus S}) - H(X^V, Y^{V \setminus S}) \) so as a function of \( S \) this is neither submodular nor supermodular (but is a difference).
Graphical Multicast Network

- We’ve got a general network $G = (V, E)$, each node is a sender-receiver pair, and each edge $(j, k) \in E$ is a noiseless communication link with capacity $C_{j,k}$.
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- A source (w.l.o.g. node 1) wants to communicate a message $W \in [2^{nR}]$ to a set of receivers $D \subseteq V$, all of whom get an estimate $\hat{W}_j$ for $j \in D$ of the message.
Graphical Multicast Network

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- Each node \( k \in [2:|V|] \) can act as a relay to help source node communicate messages to destination nodes, and general computational relay operations are allowed.
The capacity of the graphical multicast network $G = (V, E, C)$ with destination set $D \subseteq V$ is given as:

$$C = \min_{j \in D} \min_{S \subseteq V} \left\{ C(S) \right\}$$

where $C(S)$ is the cut value in the network (sum of cut edges).
Theorem 4.2 (Network Coding Theorem)

The capacity of the graphical multicast network $G = (V, E, C)$ with destination set $D \subseteq V$ is given as:

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- This is relatively easy to compute, min-cut max-flow duality.
- But many network IT instances do not have known achievable regions or capacities and this is ongoing research.
Where to go next?

- For communications, the Gamal & Kim book seems very comprehensive for network information theory.
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- For particular applications (e.g., speech, language, bioinformatics, computer vision, or others) learn lots about that application (there is no substitute).
End