BIG PICTURE

CONVEXITY

JENSTEN'S INEQUALITY
\[ f(E(X)) \leq E(f(X)) \]

APPLY TO
\[ y = q(x) \]
\[ E_{X \sim X} \log y \frac{q(x)}{P(x)} \]
\[ p = \frac{ac}{\sum ai} \]
\[ q_i = \frac{b_i}{\sum b_i} \]

Log-sum Inequality
\[ \log \sum a_i e^{x_i} \leq e^{\log \sum a_i} \frac{\sum x_i}{\sum a_i} \]

D\( (p\|q) \geq 0 \)

Apply to
\[ p = p(x,y) \]
\[ q = p(x)q(y) \]

I\( (X;Y) \geq 0 \)

H\( (X) \geq H(X|Y) \)

Reduced Entropy

D\( (p\|q) \geq 0 \)

\[ = \Rightarrow H(X) \leq \log |X| \]

D\( (p\|q) \) is common in the pairs (p,q)
STOCHASTIC PROCESS

Indexed collection of R.V.'s
X₁, X₂, ..., Xₙ

I.I.D. (i.i.d.) process
P(Xₙ) = P(X₁) ≠ m
P(X₁, X₂, ..., Xₙ) = P(X₁)P(X₂) ... P(Xₙ)

Markov Chain
P(Xₙ|Xₙ₋₁, ..., X₁) = P(Xₙ|Xₙ₋₁)

W.L.L.N.
\[ \frac{1}{n} \sum_{i=1}^{n} X_i \rightarrow E[X] \text{ w.h.p.} \]

Markov Inequality

Chebyshev Inequality

X → Y → Z

Data Processing Inequality

I(X; Y) ≥ I(X; Z)

Sufficient Statistic

\( H(X^n) = \lim_{n \to \infty} \frac{H(X^n)}{n} \)

H(XX; X⁻¹) ≥ H(X)

\[ H(X^n) ≤ 2m_{H(X)} \]

W.h.p. that \( p_{X^n}(x^n) \) typical
<table>
<thead>
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Symbol → Code → uniquely-decodable
Symbols → Code Extension

Kraft Inequality: A code is prefix code iff the
lengths li satisfy
\[ \sum_{i=1}^{n} D^{-li} \leq 1 \]

Simple Application: Devise an instantaneous code
with minimum expected
length for di symbols with
a given distr p(x).

\[ \min \sum_{i=1}^{n} p_i li \]
s.t. \[ \sum_{i=1}^{n} D^{-li} \leq 1 \]

Convex Constraints:
Convex Optimization Problem
(i) Can be solved for
exactly

Solution: Set lengths to proportional (equal) to the
sums of the symbols.

For any Instantaneous Code:

\[ H_0(x) < L < H_0(x) + 1 \]

True for Shannon's optimal codes.